TRADE AND GROWTH: A SIMPLE MODEL WITH NOT-SO-SIMPLE IMPLICATIONS

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Abstract. We present a simple model of international trade (IT) and growth. The model yields a unique equilibrium path in which the relationship between exogenous and endogenous variables does not resemble the equations estimated by the empirical literature: Ours are not linear, despite the fact that technology and demand are linear, they do not include variables used in this literature like shares of IT and investment and include variables that have never been used in this literature such as comparative and absolute advantage, specialization patterns, saving habits and technology of partner countries. Finally, the impact of the initial level of income and the number of years that the economy has been open is far more complicated than had been assumed by the literature.

1. **Introduction.** The question of the link between International Trade (IT) and growth is a popular issue outside our profession. What do we have to say about it? On the one hand we have general theoretical results on the performance of open economies that provide conditions for the convergence to a steady state and study its properties.[1]. On the other hand, the empirical literature presents conflicting views on this relationship: Some authors maintain that a positive relationship between these two variables shows up in the data. Others are skeptical about it.[2] This literature is so huge that it is impossible to summarize here, but examples of the former are [9] and [18] while examples of the latter are [13] and [17]. A characteristic of the empirical literature is the emphasis on sophisticated methods of estimation -see [19] for a recent entry- and not on deriving the estimated equations from a formal model.

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[1]See [2], [3], [5], [6],[10], [14], [15], [20], and [21]. For a survey of the results on comparative dynamics see [7]

[2]One branch of this literature uses cross-country regressions to search for linkages between growth rates and other variables. Another branch concentrates on the dynamic paths followed by a country or group of countries, especially after a major trade liberalization (TL).

This paper presents a simple model of trade and growth that attempts to start a bridge between theory and empirical studies. The only similar work is by [23].[3] Our model is a blend of a two sector Ricardian model where trade is driven by differences in technology (comparative advantage) and capital is the only factor of production, and the Harrod-Domar-Rebelo model of endogenous growth in which consumption is linear with income.[4] In these three models, in equilibrium, capital grows at a constant rate.[5]

In our model equilibrium is unique and can be of three possible kinds that we will call regimes: 1) Both countries completely specialize. 2) Only the country with a comparative advantage in consumption goods (country B) specializes. 3) Only the country with a comparative advantage in capital goods (country A) specializes. We find that the growth rate of country A is the same in the three regimes. But the growth rate of country B is different in each of these regimes. The dynamics of the model follows three possible trajectories. Each of this trajectories ends in one of the regimes, irrespective of the initial capital stock. Thus in our model, unlike Harrod-Domar-Rebelo's, there is transitional dynamics.

Our model yields predictions that look very different from the models employed by the empirical literature, namely:

- Even in the simplest cases, relationships between exogenous and endogenous variables are not linear: They are piecewise linear or multiplicative with each specialization pattern yielding a different equation.[6]

- Some variables used by the empirical literature as exogenous variables, like the shares of investment or IT in GDP, are endogenous. This point has been already noticed but the usual defense is that these variables are used as a proxy of other, non quantifiable, variables. We show that in some cases the intended relationship between any of these shares and the growth rate either does not exists or it goes in opposite direction to the one assumed before. Moreover in some cases these two variables are in a one-to-one relationship so both should not be used at the same time.

- Other variables play a role that is different from the one assumed by this literature like the initial level of income and the number of years that the economy has been open. Trade distortions -that in our case take the extreme form of autarky- affect growth in a very complicated way.

- Finally, variables that are important in our model have been neglected by the applied literature, such as comparative and absolute advantage, specialization patterns and saving habits and technology of partner countries. Once the mechanism that operates behind these variables is understood, it is difficult to think of a model where they do not play a role.

[3] In order to concentrate on the fundamentals, our model leaves aside important topics like oligopoly, asymmetric information, externalities and any possible "friction". Innovation, a topic of paramount empirical relevance, is also disregarded, see [11]. Also we only consider equilibrium states leaving aside any stability analysis of equilibrium.

[4] In [8] and [21], this follows from their assumption that consumption depends linearly on income. In [16], in which a representative consumer maximizes an intertemporal utility function, this occurs because of the special form of the utility function.

[5] An important part of the theory of IT is based on the Heckscher-Ohlin-Samuelson (HOS) model with two factors. Our model is capable of incorporating a second factor under additional assumptions on technology, see the Appendix. Moreover, [3] has shown that the long run equilibrium of the HOS model displays Ricardian features. In particular " countries specialize according to comparative advantage".

[6] The equations estimated by the empirical literature are, almost invariably, linear.

What do we learn from here? On the negative side the main lesson from our paper is that a simple model of IT and growth produces results that are far from the "common sense" equations that have been estimated. On the positive side, our model provides a deeper understanding of the mechanism of trade and growth, some cautions about those already used, and a fresh set of variables to be used. In any case, more work is necessary to produce a workable theoretical model and to test the implications.

The rest of the paper goes as follows: Section 2 presents the model, Section 3 discusses the empirical implications and Section 4 concludes. An Appendix extends the model to international capital mobility and two factors.

2. ***The model.*** We assume two countries, A and B, and two goods: a consumption good and a capital good. Capital can be understood as physical, human or social. For simplicity we will assume that capital does not depreciate. The aggregate stock of capital in country $i = A, B$ is denoted by K^i. The quantity of capital used in the production of the consumption (resp. capital) good is denoted by K^i_C (resp. K^i_I) $i = A, B$. Capital is mobile between sectors and fully utilized. Thus

$$K^i = K^i_C + K^i_I, \; i = A, B. \tag{1}$$

Capital is assumed to be immobile between countries. In an Appendix we prove that if consumer tastes are identical between countries, the main conclusions of this paper hold under international capital mobility. We assume that output is produced by means of capital alone. We may introduce a second factor, labor, whose supply (including emigrants) is arbitrarily large. Then, either wages are at subsistence level (normalized to zero), or, in each country, capital-labor ratios are identical in each sector. The later interpretation is developed in an Appendix were it is shown that our main conclusions hold under this additional assumption.

Production takes place under constant returns to scale. The production function of consumption and capital goods in country $i = A, B$ is

$$C^i_O = \beta^i_C K^i_C \quad \text{and} \quad I^i_O = \beta^i_I K^i_I, \tag{2}$$

where C^i_O (resp. I^i_O) stands for the output of consumption (resp. capital) good (hence the subindex) and β^i_I and β^i_C are the average productivity of capital in country i in the production of capital (β^i_I) and consumption (β^i_C) goods.

For international trade to be mutually advantageous and without loss of generality we assume that

$$\frac{\beta^B_I}{\beta^B_C} < \frac{\beta^A_I}{\beta^A_C}, \tag{3}$$

i.e., country A (resp. B) has a comparative advantage in the production of the capital (resp. consumption) good.

Let P^i be price of the consumption good in country i. The price of the output of the capital good is the numeraire. Let r^i be the rental price of capital in country i. We will assume that firms are price-takers. Since production takes place under constant returns to scale, profit maximization implies no positive profits. In particular if a good is produced, revenues should cover exactly its production costs. And if a good is not produced, revenues cannot exceed costs. In the sequel, subscript O refers to the output in the country in the superscript. Supply side is characterized by the following equations:

$$P^iC^i_O \leq r^iK^i_C \quad \text{or} \quad \beta^i_C P^i \leq r^i \quad \text{and if strict inequality holds} \quad C^i_O = 0. \tag{4}$$

$$I^i_O \leq r^iK^i_I \quad \text{or} \quad \beta^i_I \leq r^i \quad \text{and if strict inequality holds} \quad I^i_O = 0. \tag{5}$$

Let $Y^i = r^iK^i$ be national income in country i. When there is trade between countries we have to distinguish between output and where the goods are allocated. So let I^i_D, $i = A, B$ be the new capital goods allocated to country i and C^i_D be the demand for consumption goods in country i. This demand is assumed to be of Keynesian type, i.e., consumption is linear on real income,

$$C^i_D = \frac{c^iY^i}{P^i} \quad 0 < c^i < 1 \quad i = A, B. \tag{6}$$

In the conclusions we discuss how to obtain this function from utility maximization. Demand for investment in country i, (I^i_D) is assumed to be

$$I^i_D = Y^i - \frac{c^iY^i}{P^i}P^i = (1 - c^i)Y^i \tag{7}$$

so all income is spent either in consumption or in capital goods. Finally let g^i be the rate of growth of capital in country i defined as

$$g^i = \frac{\dot{K}^i}{K^i}$$

where $\dot{K}^i$ is the increase in the capital stock. Since there is no depreciation

$$\dot{K}^i = I^i_D.$$

As we will see, in equilibrium, all the relevant variables depend on the capital stock. And the rate of growth of any variable can be easily calculated from g^i.

Firstly let us consider that capital is given. In this case the model is a two sector Ricardian model. We solve the model for the case of autarky and for all the possibilities opened up by free trade. The dynamic paths of the economy when investment accrues the capital stock will be considered later on.

2.1. **Autarky.** In this regime we do not have to distinguish between production and demand so we drop the corresponding subscript for C and I. Equilibrium values of the variables under autarky are denoted with the superscript $*$. From equations (2), (4), (5) and (6), $i = A, B$ we obtain,

$$I^{i*} = \beta^i_I(1 - c^i)K^i. \quad C^{i*} = c^i\beta^i_CK^i. \quad r^{i*} = \beta^i_I. \tag{8}$$

$$g^{i*} = \beta^i_I(1 - c^i). \quad P^{i*} = \frac{\beta^i_I}{\beta^i_C}. \tag{9}$$

Notice that under autarky the rate of growth equals the propensity to save $(1-c^i)$ divided by the capital-output ratio $1/\beta^i_I$ of the capital goods sector. This is exactly the (warranted) rate of growth in the Harrod-Domar model.

Next, suppose that both countries open up to international trade. Now we delete the superscript of P, since now domestic prices are international prices. Finally, we introduce a new variable,

$$i^i \equiv \frac{Value\ of\ Trade}{Income\ of\ i}. \tag{10}$$

i^i measures the relative importance of trade in national income of country i.

To see all possible cases of specialization, consider Table 1 below. The first two entries in the main diagonal are impossible (marked I) because (6) requires that both goods must be produced. Any cell in which a country specializes in a good in which it has comparative *disadvantage* is marked VCA (violates comparative advantage). This leaves only three possibilities that we explore in turn. See the Appendix for the calculations.

		Country B		
		Consump. good only	Cap. good only	Both goods
Country A	Consump. good only	I	V C A	V C A
	Cap. good only	Complete Spec.	I	A specializes
	Both goods	B specializes	V C A	V C A

Table 1

2.2. **Complete specialization.** In this regime, Country A produces capital goods only and Country B produces consumption goods only. We denote equilibrium variables by an upper bar. From equations (2), (5), (6) and (10) we obtain for Countries A and B

$$\bar{I}_D^A = \beta_I^A(1-c^A)K^A. \quad \bar{C}_D^A = (1-c^B)\beta_C^B K^B. \quad \bar{r}^A = \beta_I^A. \quad \bar{\imath}^A = c^A. \tag{11}$$

$$\bar{I}_D^B = \beta_I^A c^A K^A. \quad \bar{C}_D^B = c^B \beta_C^B K^B. \quad \bar{r}^B = \frac{\beta_I^A c^A K^A}{(1-c^B)K^B}. \quad \bar{\imath}^B = 1-c^B. \tag{12}$$

From (4), (11) and (12) it is easily seen that

$$\bar{g}^A = \beta_I^A(1-c^A). \quad \bar{g}^B = \frac{\beta_I^A c^A K^A}{K^B}. \quad \bar{P} = \frac{\beta_I^A c^A K^A}{(1-c^B)K^B \beta_C^B}. \tag{13}$$

Supply equations (4) and (5) boil down to

$$\frac{(1-c^B)\beta_I^B}{c^A\beta_I^A} \leq \frac{K^A}{K^B} \leq \frac{(1-c^B)\beta_C^B}{c^A\beta_C^A}. \tag{14}$$

We call this regime *Complete Specialization*. It occurs for intermediate values of K^A/K^B.

2.3. **Country B specializes in consumption goods.** In this regime, Country A produces both goods and country B specializes in the consumption good. We denote equilibrium variables by a hat. From equations (2), (5), (6) and (10) we obtain for Countries A and B

$$\hat{I}_D^A = \beta_I^A(1-c^A)K^A. \quad \hat{C}_D^A = c^A\beta_C^A K^A. \quad \hat{r}^A = \beta_I^A. \quad \hat{\imath}^A = \frac{\beta_C^B(1-c^B)K^B}{\beta_C^A K^A}. \tag{15}$$

$$\hat{I}_D^B = \frac{\beta_C^B(1-c^B)K^B\beta_I^A}{\beta_C^A}. \quad \hat{C}_D^B = c^B K^B \beta_C^B. \quad \hat{r}^B = \frac{\beta_I^A\beta_C^B}{\beta_C^A}. \quad \hat{\imath}^B = 1-c^B. \tag{16}$$

From (4), (15) and (16) it is easily seen that

$$\hat{g}^A = \beta_I^A(1 - c^A). \quad \hat{g}^B = \frac{\beta_C^B \beta_I^A (1 - c^B)}{\beta_C^A}. \quad \hat{P} = \frac{\beta_I^A}{\beta_C^A}. \tag{17}$$

Supply equations (4) and (5) boil down to

$$\frac{K^A}{K^B} > \frac{(1 - c^B)\beta_C^B}{c^A \beta_C^A} \quad (\text{or } c^A > \hat{\imath}^A). \tag{18}$$

We call this regime *Country B Specializes.* It occurs for large values of K^A/K^B.

2.4. **Country A specializes in capital goods.** In this regime Country A specializes in the capital good and country B produces both goods. We denote equilibrium variables by a tilde. From equations (2), (5), (6) and (10) we obtain for Countries A and B

$$\tilde{I}_D^A = \beta_I^A K^A (1 - c^A). \quad \tilde{C}_D^A = \frac{c^A \beta_I^A K^A \beta_C^B}{\beta_I^B}. \quad \tilde{r}^A = \beta_I^A. \quad \tilde{\imath}^A = c^A. \tag{19}$$

$$\tilde{I}_D^B = \beta_I^B (1 - c^B) K^B. \quad \tilde{C}_D^B = c^B \beta_C^B K^B. \quad \tilde{r}^B = \beta_I^B. \quad \tilde{\imath}^B = \frac{c^A \beta_I^A K^A}{\beta_I^B K^B}. \tag{20}$$

From (4), (19) and (20) it is easily seen that

$$\tilde{g}^A = \beta_I^A(1 - c^A). \quad \tilde{g}^B = \beta_I^B(1 - c^B). \quad \tilde{P} = \frac{\beta_I^B}{\beta_C^B}. \tag{21}$$

Supply equations (4) and (5) boil down to:

$$\frac{(1 - c^B)\beta_I^B}{c^A \beta_I^A} > \frac{K^A}{K^B} \quad (\text{or } 1 - c^B > \tilde{\imath}^B). \tag{22}$$

We call this regime *Country A Specializes.* It occurs for small values of K^A/K^B. It is the only regime in which growth rates of both countries equal those under autarky.

The previous results are summarized in the following two propositions. Proposition 1 spells the conditions of occurrence of the three regimes under free trade.

Proposition 1. *a) When the capital in Country A is small in relationship to the capital in Country B, Country A specializes in the production of capital goods and Country B produces both goods. Mathematically*

$$\frac{(1 - c^B)\beta_I^B}{c^A \beta_I^A} > \frac{K^A}{K^B}$$

b) When the capital in Country A is large in relationship to the capital in Country B, Country B specializes in the production of consumption goods and Country A produces both goods. Mathematically

$$\frac{K^A}{K^B} > \frac{(1 - c^B)\beta_C^B}{c^A \beta_C^A}$$

c) In the remaining case, both countries specialize in the production of the good they have comparative advantage. Mathematically

$$\frac{(1 - c^B)\beta_I^B}{c^A \beta_I^A} \le \frac{K^A}{K^B} \le \frac{(1 - c^B)\beta_C^B}{c^A \beta_C^A}$$

Proposition 1 says that when a country has a large capital relative to the other country, in the free trade equilibrium this country specializes in the production of the good it has comparative advantage. When neither country has a large relative capital, both specialize.

The next proposition focus on the growth rates.

Proposition 2. *a) In all regimes, g^A is constant and equal to $\beta_I^A(1-c^A)$.*
b) Under complete specialization, g^B equals $\beta_I^A c^A K^A/K^B$ so is linearly increasing on K^A/K^B.
c) In any other regime, g^B is constant. It equals the autarkic growth rate when Country A Specializes and is larger than the autarkic rate in the other regimes.

Part a) of Proposition 2 is reminiscent of a classical result in von Neumann-Leontief-Sraffa models that if the economy can be partitioned such that there are sectors whose input-output matrix is indecomposable, the common rate of growth of these sectors can be calculated using the technology of these sectors only. It implies that for the country with comparative advantage in the production of capital, free trade has no impact on its growth rate: it only has a level effect. Parts b) and c) imply that the impact of free trade on the growth of the country with a comparative advantage in consumption goods is not straightforward. It may increase the growth rate or it may have only a level effect. In particular, if free trade makes the other country to specialize, free trade has no effect on growth.

We now turn our attention to the consequences of capital accumulation under free trade. We have three possibilities which we study in turn.

2.5. **The growth rate of A is always greater than the growth rate of B.** i.e. $g^A > g^B$ everywhere. This occurs iff

$$\beta_I^B(1-c^B) < \beta_I^A(1-c^A) \text{ and } \beta_C^B(1-c^B) < \beta_C^A(1-c^A)$$

or

$$\frac{1-c^A}{1-c^B} > \max\{\frac{\beta_I^B}{\beta_I^A}, \frac{\beta_C^B}{\beta_C^A}\} = \frac{\beta_C^B}{\beta_C^A} \tag{23}$$

where the last equality follows from (3). The left hand side can be interpreted as the advantage of country A in savings and the right hand side is the advantage of country B in the production of the consumption good. The interpretation is that country B has a low advantage in capital accumulation and in the production of the consumption good (which is the good in which country B has comparative advantage). In this case country B grows at a lower pace than country A. If tastes are identical in both countries, this case occurs iff $\beta_C^A > \beta_C^B$. Thus in our model, absolute advantage may matter. Note that this case implies that in autarky, A grows faster than B. We will refer to this case as "*A Grows Faster than B*".

In Figure 1, we have pictured this case in which for $K_A/K_B \in [0,2]$ only country A specializes, for $K_A/K_B \in [2,4]$ both countries specialize and for $K_A/K_B > 4$ only country B specializes. The dashed (resp. solid) line is A's (B's) growth rate.

2.6. **The growth rate of B is always greater than the growth rate of A.** i.e. $g^B > g^A$ everywhere. This occurs iff

$$\beta_I^B(1-c^B) > \beta_I^A(1-c^A) \text{ and } \beta_C^B(1-c^B) > \beta_C^A(1-c^A)$$

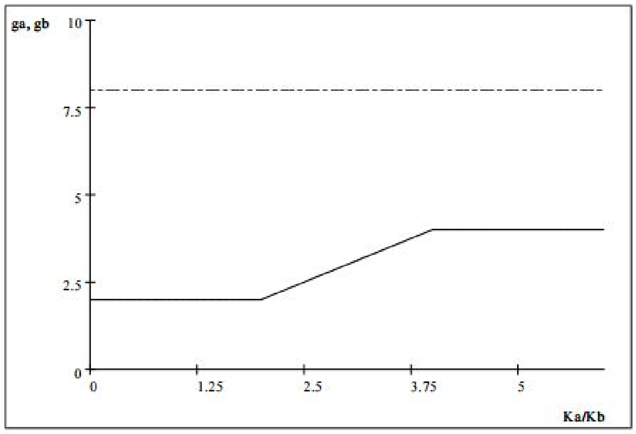

FIGURE 1.

or

$$\frac{1-c^A}{1-c^B} < \min\{\frac{\beta_I^B}{\beta_I^A}, \frac{\beta_C^B}{\beta_C^A}\} = \frac{\beta_I^B}{\beta_I^A} \tag{24}$$

This is the polar case to the previous one and it has a similar interpretation. In particular this case implies that in autarky, B grows faster than A. We will refer to this case as "*B Grows Faster than A*".

In Figure 2, we have pictured this case for the same values of the specialization intervals as in Figure 1. The dashed (resp. solid) line is A's (B's) growth rate.

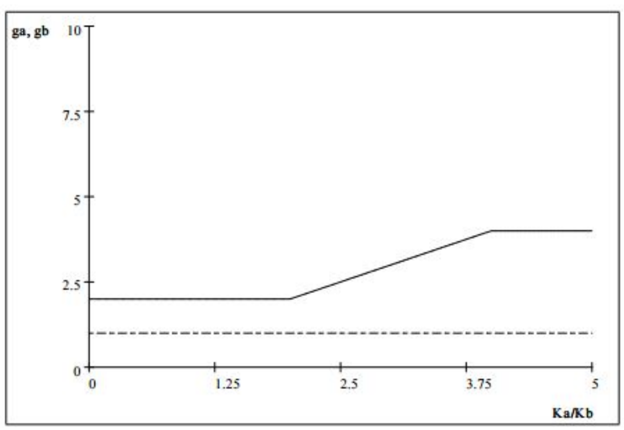

FIGURE 2.

2.7. **The growth rate of A is sometimes greater, sometimes smaller than the growth rate of B.** This case occurs iff

$$\beta_I^B(1-c^B) < \beta_I^A(1-c^A) \text{ and } \beta_C^B(1-c^B) > \beta_C^A(1-c^A)$$

or

$$\frac{\beta_I^B}{\beta_I^A} < \frac{1-c^A}{1-c^B} < \frac{\beta_C^B}{\beta_C^A} \tag{25}$$

In this case the advantage of country A in savings is between the relative advantages of countries A and B. If tastes are identical in the two countries, this case occurs when country A (resp. B) has absolute advantage in the production of capital (consumption) goods. We will refer to this case as "*Growth of A and B can go either way*".

In Figure 3, we have pictured this case for the same values of the specialization intervals as in Figures 1 and 2. The dashed (resp. solid) line is A's (B's) growth rate. The intersection of both growth rates is at $(1-c^A)/c^A$. In this point, the ratio of capitals equals the ratio of exports of A and B.

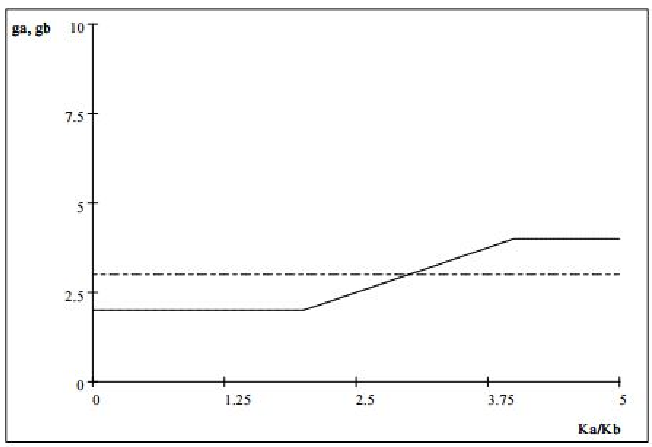

FIGURE 3.

Our next result studies the dynamic trajectory of the capital ratio and the specialization patterns. We show that countries always end in one specialization pattern and that this pattern does not depend on the initial condition.

Proposition 3. *Starting from any initial value of* K^A/K^B*:*

a) When "Growth of A and B can go either way", $K^A/K^B \to (1-c^A)/c^A$ *and, in the limit, both countries specialize. The rate of growth of B converge to the growth rate of A, namely* $\beta_I^A(1-c^A)$.

b) When "A Grows Faster than B", $K^A/K^B \to \infty$ *and, in the limit, country B specializes. The difference between growth rates decreases in the complete specialization regime but it is constant in the other two regimes.*

c) When "B Grows Faster than A" $K^A/K^B \to 0$ *and, in the limit, country A specializes. The difference between growth rates decreases in the complete specialization regime and it is constant in the other two regimes.*

Proof. a) Firstly, suppose that country B specializes. But there,

$$\hat{g}^A = \beta_I^A(1-c^A) < \hat{g}^B = \frac{\beta_C^B \beta_I^A(1-c^B)}{\beta_C^A}.$$

Thus K^A/K^B decreases and we end up in the regime of complete specialization.

Secondly, suppose that country A specializes, $\tilde{g}^A = \beta_I^A(1-c^A) > \tilde{g}^B = \beta_I^B(1-c^B)$. Thus, K^A/K^B grows and we will end up in the regime of complete specialization.

Finally if both countries specialize and $\bar{g}^A < \bar{g}^B$ the rate of growth of country B ($= \beta_I^A c^A K^A/K^B$) decreases up to the point in which it equals the rate of growth of country A. If $\bar{g}^A > \bar{g}^B$, the opposite occurs. Thus,

$$\bar{g}^A = \beta_I^A(1-c^A) = \bar{g}^B = \frac{\beta_I^A c^A K^A}{K^B} \text{ and } \frac{K^A}{K^B} = \frac{1-c^A}{c^A}.$$

b) Notice that here, $g^A > g^B$. Thus K^A/K^B increases with time and, no matter the initial value of K^A/K^B is, we end up with country B specialized. In this regime the rate of growth of capital of country B is the largest among all equilibrium rates.
c) Virtually identical to b) above. □

The proof of Proposition 3 follows essentially from a look to Figures 1 (for part b)), 2 (for part c)) and 3 (for part a)).

From the proof of Proposition 3 we see that the following trajectories are possible:

1. B Specializes $\Rightarrow$ Complete Specialization. This is the first case in part a) of the proof of Proposition 3..
2. A Specializes $\Rightarrow$ Complete Specialization. This is the second case in part a) of the proof of Proposition 3.
3. A Specializes $\Rightarrow$ Complete Specialization $\Rightarrow$ B Specializes. This is part b) of the proof of Proposition 3.
4. B Specializes. $\Rightarrow$ Complete Specialization $\Rightarrow$ A specializes. This is part c) of the proof of Proposition 3.

The first and the third possibilities generate a growth rate for country B that is constant, then increasing and then constant and the second and the fourth possibilities generate a growth rate for B that is constant, then decreasing and then constant. Any part of these trajectories are possible too. Finally note that, depending on the parameters of the economy, any regime may be the limit of the accumulation process or it may send the capital ratio to the neighbor regime.

3. **Empirical Implications.** In this section we highlight the implications of our model and compare with the set up employed by some papers of the empirical literature.

1. Functional forms. There is not a *single functional form* that relates exogenous variables to growth rates: Country A always grows at the same rate, under autarky or under any specialization pattern. In country B, functional forms depend on specialization patterns and trading opportunities. And when functional forms are stable, switching from autarky to free trade has no effect on growth.

Equations are not linear despite the linearity assumed throughout the model. They are piecewise linear or multiplicative so they are captured better using logarithms.

2. Variables used. The following variables have been linked to growth by the empirical literature:
i): Share of IT in GDP.
ii): Investment share.
iii): Exchange Rate and/or Trade distortions.

iv): Initial level of Income.

v): Number of years that the economy has been open.

We see immediately the problem with i): *The share of IT in GDP is not an independent variable*: In other words, both the share of IT and the growth rate depend on the fundamentals of the economy but there is no causation at all between them. If this variable is used as a proxy, notice that under complete specialization or if A specializes $g^A = \beta_I^A(1-i^A)$ -so we get a negative relationship!- and if B specializes $\hat{g}^A$ is independent of i^A and i^B. In the case of B, $\bar{g}^B = \beta_I^A \bar{\imath}^A K^A/K^B$, $\hat{g}^B = \beta_C^B \beta_I^A \hat{\imath}^B/\beta_C^A$ and $\tilde{g}^B$ is independent of i^A and i^B. Thus, our model does not support the view that to introduce the share of IT in GDP in the equations to be estimated is a good idea.

The problem with ii), the investment share -which equals $1-c^i$- is that under complete specialization $B's$ rate of growth does not depend on $1-c^B$, but positively on c^A! Thus if we have in our sample many countries completely specialized in consumption goods, the relationship between g^i and $1-c^i$ will be blurred.

iii) measures how changes in trade barriers affect growth. In our model these changes take the simple form of switching from autarky to free trade. Despite this simplification *this effect can not be captured by an additive dummy variable*: Growth in country A is unaffected (the only effect is a level effect) and, in country B, the following possibilities may arise after a Trade Liberalization (TL):

- *No effect at all.* This occurs if after TL A specializes and 8 holds.

- *Level effect only*: This occurs if after a TL A specializes and 5-7 hold. In this case g^B increases and remains constant thereafter.

- *No effect in the short run. Acceleration until a certain point later on.* This occurs if after a TL A specializes and 5 holds. Under 6 both growth rates converge and under 7 they just approach each other.

- *Acceleration in the short run and constancy thereafter.* This occurs if after a TL both countries specialize and 5 holds. Under 6 both growth rates converge and under 7 they just approach each other.

- *Level effect, deceleration and growth like in autarky later on.* This occurs under 8 if after a TL either B specializes or both countries specialize. In the first case the new growth rate is maintained for a while.

- *Level effect, deceleration and convergence.* This occurs under 5 and 6 if after a TL either B specializes or both countries specialize.

iv), the initial level of income is inversely related in neoclassical models to the growth rate. Here for country B it is related in a piecewise linear way with relative capital stocks that determine specialization patterns. Clearly, initial level of income is an imperfect measure of relative capital stocks.

v), the number of years that the economy has been open, determines in our model the switch between specialization patterns. But the relationship between the latter and the growth rate depends on the relative initial capital stock -that determines the initial position- and growth rates of past periods -that determines the amount of time that a country spends in a given specialization pattern.

3. Variables omitted. We see that rates of growth depend on variables that, to the best of my knowledge, do not appear in the empirical literature like:

I) Comparative advantage in the production of capital or consumption goods.

II) Relative stock of capital and specialization patterns.

III) Saving habits and technology of partner countries.

It is unlikely that such variables could be absent in more complicated models. In fact, it is surprising that such variables have been forgotten because their role is clear: I) Countries with comparative advantage in goods that make the economy grow, behave differently than countries with no such advantage. For instance, the former countries are less likely to be affected by opening to trade or by tariffs. II) Relative size matters, because this size shapes specialization patterns that, in turn, explains supply. III) When development is demand oriented and capital goods are imported and paid with exports of consumption goods, characteristics of trade partners are important. And specialization patterns depend in the long run on relative growth rates.

4. ***Conclusions.*** In this paper we show that even in a simple model of international trade where all relationships in the basic economy are linear, the exact relationship between trade and growth cannot be captured by a single equation.[7] It is clear that a full assessment of the impact of trade on growth needs a complicated model with general assumptions on factors, technology, etc., and where imperfections of competition and information play an important role. But it is unlikely that in such a model the relationship between growth and other variables would be simpler than in our model.

A possible criticism of our paper is that our assumption of a constant fraction of income devoted to savings is not based on utility maximization. When the economy is always completely specialized there is a utility function that provides microfoundations to this assumption (see [4] p. 531). But when the economy moves between specialization patterns, it is not clear which preferences yield such an assumption. In any case, most of our remarks in Section 3 are still valid with a variable saving rate because they remain applicable for *any* given savings rate. Predictions of a trade liberalization still hold as long as there is no switch from one specialization pattern to another. In any case, if the linear relationship between consumption and income is lost we should expect that the relationship between trade and growth becomes even more complicated.

Another criticism of our paper is that an important part of the theory of international trade is based on the Heckscher-Ohlin-Samuelson model with two factors and a differentiable production function. But our model is capable of incorporating a second factor under additional assumptions on technology, see Appendix. Again, more general assumptions will produce even more complicated models. Moreover, [3] (p. 713) has shown that the long run equilibrium of the Heckscher-Ohlin-Samuelson model displays Ricardian features. In particular "countries specialize according to comparative advantage". Finally, such a model produces very often only level effects (see [14] p. 12). Our model is capable of producing level *and* growth effects.

What do we learn from our exercise other than the complexity of the issue? On this count our model suggests, at least, three things.

1. The relevance of variables not considered so far like comparative and absolute advantage, relative capital stock and consumption habits and technology of partner countries.
2. Some variables used so far, like the trade share, the savings rate, etc. do not play the role that they were assumed to play.
3. Logarithmic and piecewise linear functional forms may capture the intended relationships better.

[7] See [1] for a similar point based in the european experience.

We hope that our paper encourages other researchers to build models of growth and international trade (for instance based on the Solow model) and explore their properties

5. *Appendix.*

5.1. The model with mobile capital between countries.

Assume that capital flows from the country with the lowest rental rate to the country with the highest rental rate. We will see that the dynamic analysis presented in Proposition 3 still valid if $c^A = c^B$. This condition corresponds to the assumption often made in International Trade that tastes are identical in both countries. Recall that:

$$\text{Under complete specialization, } \bar{r}^A = \beta_I^A \text{ and } \bar{r}^B = \frac{\beta_I^A c^A K^A}{(1-c^B)K^B}. \tag{26}$$

$$\text{If country B specializes, } \hat{r}^A = \beta_I^A \text{ and } \hat{r}^B = \frac{\beta_I^A \beta_C^B}{\beta_C^A}. \tag{27}$$

$$\text{If country A specializes, } \tilde{r}^A = \beta_I^A \text{ and } \tilde{r}^B = \beta_I^B. \tag{28}$$

There are three possible cases:

- $r^A > r^B$ **for all** K^A/K^B. In this case, capital flows from B to A. (27) implies that $\beta_C^A > \beta_C^B$. Notice that if $c^A = c^B$ this is just (23). There $g^A > g^B$ without capital mobility, so this tendency is reinforced by capital mobility.

- $r^A < r^B$ **for all** K^A/K^B. In this case capital flows from A to B. (27) implies that $\beta_I^A < \beta_I^B$. Notice that if $c^A = c^B$ this case is just (24) in the main text. In this case $g^B > g^A$ without capital mobility, so this tendency is reinforced by capital mobility.

- $r^A >$ **(resp. ¡)** r^B **for some values of** K^A/K^B**.** This case arises when $\beta_I^A < \beta_I^B$. Notice that if $c^A = c^B$ this case is identical to (25) in the main text. For $K^A/K^B >$ (resp. $<$) $(1-c^A)/c^A$, $r^B >$ (resp. ¡) r^A, so capital flows from B (resp. A) to A (resp. B). These conditions match exactly those in the model without capital mobility.

5.2. The model with a second factor.

If output is produced by capital and labor (denoted by L), supply equations read $P^i C_O^i \leq \pi^i K_C^i + w^i L_C^i$ and $I_O^i \leq \pi^i K_I^i + w^i L_I^i$, where now π^i (resp. w^i) stands for the rental price of capital (resp. wages) in country i. Let l_C^i (resp. l_I^i) be the labor/capital ratio in country i in sector C (resp. I). Thus, $L_C^i \equiv l_C^i K_C^i$ and $L_I^i \equiv l_I^i K_I^i$. If we assume that $l_I^i \equiv l_C^i \equiv l^i$, supply equations read $P^i C_O^i \leq (\pi^i + w^i l^i) K_C^i$ and $I_O^i \leq (\pi^i + w^i l^i) K_I^i$. By redefining $r^i \equiv (\pi^i + w^i l^i)$ we obtain the equations for the model without labor. The model yields a linear wage-profit frontier, familiar to the researchers of linear models. It is customary to close these models by assuming some kind of bargaining between capitalist and workers.

5.3. Derivation of the equations in Sections2.2, 2.3, and 2.4.

SUBSECTION 2.2. COMPLETE SPECIALIZATION From (4) and (5) we obtain that $\beta_C^B P = r^B$ and $\beta_I^A = r^A$.

From (2), consumption output $= \beta_C^B K^B$

From (6), consumption demand =

$$\frac{c^A Y^A}{P} + \frac{c^B Y^B}{P} = \frac{c^A r^A K^A}{P} + \frac{c^B r^B K^B}{P} = \frac{c^A \beta_I^A K^A}{P} + \frac{c^B \beta_C^B P K^B}{P} = \frac{c^A \beta_I^A K^A}{P} + c^B \beta_C^B K^B = \frac{c^A \beta_I^A K_I^A}{P} + c^B \beta_C^B K_C^B.$$

And since consumption supply = consumption demand

$$\begin{aligned}
\beta_C^B K^B &= \frac{c^A \beta_I^A K^A}{P} + c^B \beta_C^B K^B \\
\beta_C^B K^B (1 - c^B) &= \frac{c^A \beta_I^A K^A}{P} \Rightarrow P = \frac{c^A \beta_I^A K^A}{\beta_C^B K^B (1 - c^B)}
\end{aligned}$$

$$r^B = \beta_C^B P = \beta_C^B \frac{c^A \beta_I^A K^A}{\beta_C^B K^B (1 - c^B)} = \frac{c^A \beta_I^A K^A}{K^B (1 - c^B)}$$

$$\begin{aligned}
C_D^A &= \frac{c^A \beta_I^A K^A}{P} = \frac{c^A \beta_I^A K^A}{c^A \beta_I^A K^A} \beta_C^B K^B (1 - c^B) = \beta_C^B K^B (1 - c^B) \\
C_D^B &= c^B \beta_C^B K^B.
\end{aligned}$$

From (7) for country A

$$I_D^A = \beta_I^A K^A - \beta_C^B K^B (1 - c^B) \frac{c^A \beta_I^A K^A}{\beta_C^B K^B (1 - c^B)} = \beta_I^A (1 - c^A) K^A.$$

And for country B

$$I_D^B = \frac{c^A \beta_I^A K^A}{K^B (1 - c^B)} K^B - c^B \beta_C^B K^B \frac{c^A \beta_I^A K^A}{\beta_C^B K^B (1 - c^B)} = c^A \beta_I^A K^A.$$

Finally (4) and (5) imply that

$$\begin{aligned}
\beta_C^A P &\leq r^A & \beta_I^B &\leq r^B \\
\beta_C^A \frac{c^A \beta_I^A K^A}{\beta_C^B K^B (1 - c^B)} &\leq \beta_I^A & \beta_I^B &\leq \frac{c^A \beta_I^A K^A}{K^B (1 - c^B)} \\
\frac{K^A}{K^B} &\leq \frac{\beta_C^B (1 - c^B)}{\beta_C^A c^A} & \frac{\beta_I^B (1 - c^B)}{c^A \beta_I^A} &\leq \frac{K^A}{K^B}.
\end{aligned}$$

SUBSECTION 2.3. B SPECIALIZES IN CONSUMPTION GOODS ,
From (4) and (5) we obtain

$$\beta_C^A P = r^A, \quad \beta_I^A = r^A, \quad P = \frac{\beta_I^A}{\beta_C^A}, \quad \beta_C^B P = r^B, \quad r^B = \frac{\beta_C^B \beta_I^A}{\beta_C^A}.$$

From the definition of national income,

$$Y^A = r^A K^A = \beta_I^A K^A, \quad Y^B = r^B K^B = \frac{\beta_C^B \beta_I^A K^B}{\beta_C^A}.$$

From (7),

$$I_D^A = \beta_I^A (1 - c^A) K^A, \quad I_D^B = (1 - c^B) Y^B = \frac{(1 - c^B) \beta_C^B \beta_I^A K^B}{\beta_C^A}.$$

From (6)

$$C_D^A = \frac{c^A Y^A}{P} = c^A \beta_C^A K^A, \quad C_D^B = \frac{c^B Y^B}{P} = c^B \beta_C^B K^B.$$

SUBSECTION 2.4. A SPECIALIZES IN CAPITAL GOODS From (4) and (5) we obtain

$$\beta_I^A = r^A, \quad \beta_I^B = r^B, \quad \beta_C^B P = r^B, \quad P = \frac{\beta_I^B}{\beta_C^B}.$$

From the definition of national income,

$$Y^A = r^A K^A = \beta_I^A K^A, \quad Y^B = r^B K^B = \beta_I^B K^B.$$

From (7)

$$I^A = (1 - c^A)\beta_I^A K^A, \quad I^B = (1 - c^B)\beta_I^B K^B.$$

From (6)

$$C^A = \frac{c^A Y^A}{P} = \frac{c^A \beta_I^A K^A \beta_C^B}{\beta_I^B}, \qquad C^B = \frac{c^B Y^B}{P} = c^B K^B \beta_C^B.$$

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